RADLEY COLLEGE
Entrance Scholarships

MATHEMATICS I

Thursday 12th February 2004
Time allowed 90 minutes

*You may try the questions in any order and
are not expected to complete them all.*

Show all working.

1. (No calculating aids are to be used in this question)
    a) Work out exactly

    i) 5.07×63.8

    ii) $26.197 \div 6.7$

    b) Give the answers to the following as fractions in their simplest form

    i) $\dfrac{2}{7} + \dfrac{8}{21}$

    ii) $4\dfrac{1}{6} \times 5\dfrac{2}{5}$

    iii) $\left(7\dfrac{1}{3} - 5\dfrac{3}{4}\right) \div 2\dfrac{3}{8}$

    c) Give the answers to the following in standard form.

    i) $\left(4 \times 10^6\right) - \left(3 \times 10^5\right)$

    ii) $\left(2 \times 10^{-4}\right) \times \left(7 \times 10^{-3}\right)$

    iii) $\left(9.1 \times 10^2\right) \div \left(7 \times 10^5\right)$

2. (No calculating aids are to be used in this question)

Work out as simply as possible

   a) $587^2 - 413^2$

   b) $(87 \times 46) - 46^2 + (46 \times 59)$

   c) $(84 \times 32) + (16 \times 53) + (84 \times 52) - (69 \times 16)$

   d) $\dfrac{(69 \times 431) + 431^2}{125 \times 43.1}$

3. a) Multiply out and simplify

   i) $(3x + y)(x - 2y)$

   ii) $(2x - y)(4x^2 + 2xy + y^2)$

   b) Factorise fully

   i) $18a^2b - 30ab^2$

   ii) $4x^2z - 9y^2z$

   iii) $x^2 - 3x - 40$

   c) Simplify

   i) $\dfrac{5x^2}{15x + 25x^3}$

   ii) $x^3y \div \left(\dfrac{y^3}{x^2}\right)$

4. Solve each of these equations for x

    a) $2(4x+3)-3(x+4)=14$

    b) $3x^2+16=43$

    c) $\dfrac{54}{x-3}=\dfrac{78}{x-3}-4$

    d) $(x+6)^2-(x+5)(x-2)=91$

5. Rearrange each of the following formulae to make x the subject

    a) $a+bx=c$

    b) $\dfrac{x-a}{b}=\dfrac{x-c}{d}$

    c) $\sqrt{x^2-a^2}=b$

6.   Matthew Matics has been put in charge of organising the tickets for the first performance in the new school theatre. Each ticket is to be charged at the same price as any other and the total sales are to exactly balance the cost of the production. A week before the performance, Matthew discovers that the design for the theatre has 50 more seats in it than he had been told, so he reduces each ticket price by £1 to create exactly the same income. If the number of seats he was told in the original plan was x, and the price he thought he had to charge per ticket was £y, write down an equation based on the income from ticket sales and show that it simplifies to

$$x=50y-50$$

(You may assume that all seats offered for sale are sold.)
If the total cost of the production is £1000, write down a second equation in x and y, and solve the two equations to find x and y.

7.
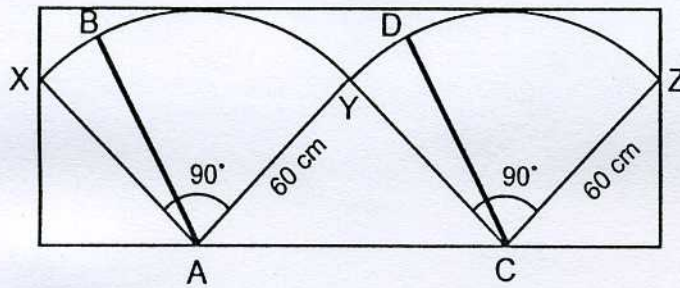
The diagram represents the front windscreen of a car, with windscreen wipers AB and CD which rotate about A and C respectively to wipe the sectors AXY and CYZ. In the central position, the wipers just reach to the top of the screen. The central position bisects angles XAY and YCZ. The wipers move so that AB and CD are always parallel.
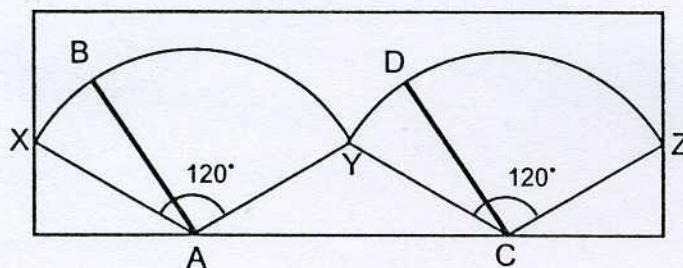
a)   In this initial design, angle XAY (and YCZ) is 90°, and the wipers are 60 cm. long.

  (i) What is the height of the screen?

  (ii) What is the width of the screen?

  (iii) What area of screen is wiped?

b)   In a second design **for the same windscreen**, the wipers are shortened so that angle XAY (and YCZ) is now 120°, as shown in the diagram.

(i) Calculate the length of the shortened wipers.

(ii) What area of screen is wiped?

8. In this question,
$$a * b = a + b - ab$$
so for example,
$$2 * 5 = 2 + 5 - 2 \times 5 = -3$$

a) Work out (i) $3 * 4$

   (ii) $(3 * 4) * 6$

b) Solve the following equations

   (i) $7 * x = -29$

   (ii) $x * x = -8$

c) Copy and complete this table.

*	0	1	2
0			
1			1
2			